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MATHEMATICAL MODELING AND ANALYSIS OF LIQUID PROPULSION SYSTEMS PROPELLANT TANK PRESSURIZATION

Viatcheslav Naoumov,
Kazan State Technical Tupolev University. K.Marx - str., 10,
420111, Kazan, Russia. E-mail: naumov@kai.ru
Mario Niwa,
CTA/IAE, Pça. Mal. Eduardo Gomes, 50 – V.Acácias – CEP 12228-904
São José dos Campos – SP; E-mail niwa@iconet.com.br
Viktor Kriukov
UNIJUÍ – Universidade Regional do Noroeste do Estado do Rio Grande do Sul
Rua São Francisco, 501- Cx. Postal 560 – CEP 98700-000 – Ijuí – RS

Abstract: *Pressurization of propellant tank of a satellite launch vehicle includes several aerothermochemical processes inside the tank, both in gas and liquid phases. In this work, a universal mathematical model for propellant tank pressurization was developed for analyzing the application of chemical non-equilibrium composition of combustion products as the pressurizing gas. In the numerical analysis, effects of the chemical reaction, the pressurizing gas injection scheme and the pressurizing gas injection velocity on pressure and average temperature were studied.*

Key words: *Liquid propulsion, Tank pressurization, Propellant tank, Rocket engine, Combustion product*

1. INTRODUCTION

It is a well-known fact that liquid propulsion systems (LPS) propellant tank pressurization involves several different aerothermochemical processes in tank's free volume and liquid propellant (Kudriatsev, 1983; Kozlov *et al.*, 1988, Huzel & Huang, 1992). Experimental tests of real pressurization systems are very expensive and thus, the number of tests should be minimum. Creation of engineering mathematical models and software which can provide simulation, analysis and optimization of parameters of LPS pressurization are, therefore, relevant step in the development activities.

The main task of pressurization is to provide the stability of tank's walls and guarantee steady LPS propellant supply during the expulsion of propellant from the tank. In the course of expulsion, the gas is injected into the free volume of tank. There are three main variants of pressurization (Huzel & Huang, 1992):

- the injection of combustion products which are produced by solid or liquid propellant gas generators (hot pressurization);
- the injection of inert gases from special storage tank (for cryogenic propellant specially);
- the injection of fuel into oxidant tank or the injection of oxidant into fuel tank (for hypergolic propellant specially) to make the chemical interaction and obtain the pressurized gas in the free volume of the tank directly (chemical pressurization).

The temperature of pressurized gas must not be very high in order to prevent the overheat of tank's walls and the overheat of the top layers of liquid propellant. In the same time the temperature of gas must not be very low in order to provide the required values of the tank's pressure using possible minimum quantity of gas.

Liquid propellant tank pressurization, involves complex phenomena, the most important are (Fig.1):

- the flow of injected gas within the free volume (ullage) of the tank;
- gas heat exchange with the tank' walls;
- the stratification of the temperature of gas in tank's volume;
- the heating of the top layers of liquid propellant;
- the vaporization of liquid propellant;
- the condensation of some species of gas on the surface of liquid propellant and on the surface of tank's walls;
- the influence of Archimedean forces upon the gas flow;
- the motion and mixing of top layers of liquid propellant;
- the creation of waves on the surface of propellant;
- the chemical interaction of gases in the free volume of the tank.

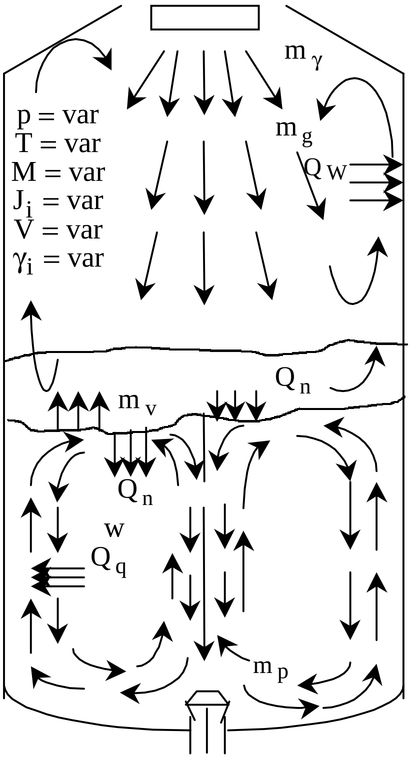

Figure 1- The scheme of aerothermochemical processes in the tank.

It can be seen, therefore, that the problem of simulation of pressurization process is very complex. It is necessary to prioritize to choose the main processes, and to make a lot of assumptions in order to create acceptable engineering mathematical model.

2. PHYSICAL SCHEME AND MAIN ASSUMPTIONS

In order to simplify the mathematical model, following main assumptions have been made:

- the motion and mixing of layers of liquid propellant are absent;
- the waves on the surface of liquid fuel are absent;
- the condensation of gas species on the wall's surface and on the surface of the propellant is absent;
- the turbulence of gas flow is absent (because the velocity of the gas in free volume is not so high);
- the equation of perfect gas is acceptable for the mathematical model of gas state in the free volume of the tank.

Several experimental investigations show the great influence of the process of stratification of gas mixture in the free volume upon the working parameters of system of pressurization (Van Dresar, 1995, for example). The reason of stratification is Archimedean force. It leads to the appearance of the mixing gas in the top part of tank and to the appearance of stratified zone (stratification of the temperature, specially) in the bottom part of the tank. Therefore, in this work, the presence of two main zones in gas volume of tank have been assumed to create more real mathematical description: mixing zone (zone 1) and stratified zone (zone 2). The boundary between the mixing and stratified zones depends on the depth of penetration of injected gas into the free volume of the tank (Fig. 2).

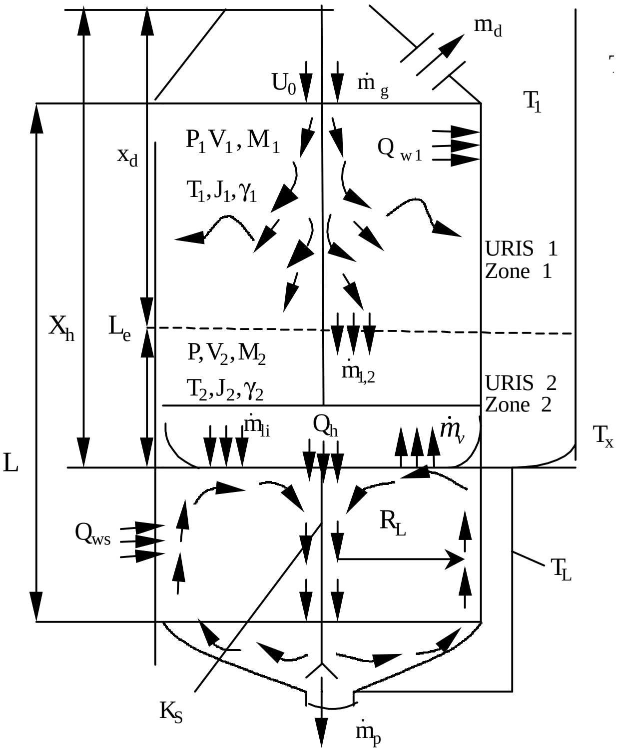

Figure 2- The calculation scheme of processes.

It is a well known fact that hot pressurization of high-boiling propellants is followed by the different chemical reactions in the gas volume of the tank. Usually, the ranges of temperature of pressurized gas is not so high (approximately 600 to 900 K). In this case, the concept of chemical equilibrium is not acceptable for the mathematical model of chemical interaction of gas mixture in free volume and therefore, it is reasonable to adopt the model of “Unsteady Reactor of Ideal Stirring-URIS” (Dregalin *et al.,* 1985, Alemasov *et al.*, 1989, Kriuokov *et al*., 1997).

URIS is a finite volume which consists of mixture of reacted species. This volume has mass and heat exchange with surroundings. An input of some initial species and an output of the products of reaction can take place. The flow of input mixture and its chemical composition is not equal to those of output mixture. Therefore, all parameters of working substance and the volume of reactor are time dependents. It is supposed here that an input mixture immediately mixes with products that are placed in the reactor. Of course, one URIS is convenient only for making of zero-dimension mathematical models, therefore the scheme of the system of URIS was proposed. Each reactor of this system has mass and heat exchange with neighboring reactors and with surroundings. This scheme was named as "System of URIS".

Any chemical reaction process in each reactor can be modeled by the set of reactions using the form:

$$\sum_i \nu'_{ij} A_i = \sum_i \nu''_{ij} A_i \qquad i = 1,....,n; \qquad j = 1, ...,m \tag{1}$$

where ν'_{ij} and ν''_{ij} - are stoichiometric coefficients for specie i in reaction j.

The scheme of two reactors have been proposed to make the description of processes in gas volume of the tank (Fig. 2). Zone of mixing (zone 1) is simulated by the URIS 1 and the stratified zone (zone 2) by the URIS 2. The mass exchange for URIS 1 is simulated by the convective mass exchange of: the output of gas mixture from the zone 1 into zone 2 during the decrease of the propellant level, the output of gas mixture through the drain valve, and the input of hot gas mixture from the injector.

The mass exchange for the URIS 2 is simulated by the convective mass exchange of: the input of gas mixture from zone 1, the input of vapors of liquid component from the surface of the propellant, and the output of condensed phase from the volume on the surface of propellant.

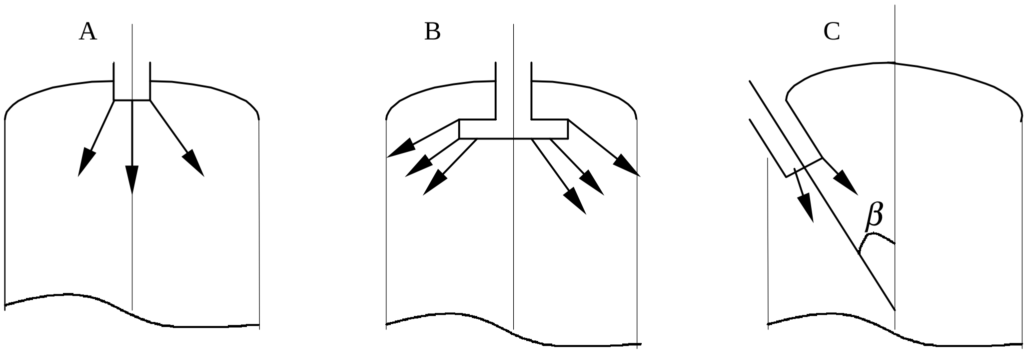

Figure 3- Main schemes of injection.

The following processes have been taken into account: heat exchange between pressurized gas and tank’s walls, heat exchange between pressurized gas and propellant and vaporization and heating of propellant.

The flow of pressurizing gas has a great influence upon the heat and mass transfer in the tank. Besides, there are several different types of gas injectors. The analysis of different constructions of injectors gave the chance to combine different systems of injection in three main schemes (Fig. 3): axial injection (a), near by walls injection (b), and cross injection (c).

3. MATHEMATICAL MODEL

The original form of the equations of chemical kinetics have been obtained for the URIS in Dregalin *et al.* (1985). Using these equations one can write the balance equations of chemical composition of gas mixture for first and second zone of tank's free volume:

$$\begin{aligned}\frac{d\gamma_{iz}}{d\tau} = - \exp(\gamma_{iz}) \sum_j \nu_{ij} k_j \left(\frac{P}{RT}\right)^{f_j} \exp\left(-\sum_p n_{pj}\gamma_{pz}\right) + \\ + \sum_q \sum_j \nu_{qj} k_j \left(\frac{P}{RT}\right)^{f_j} \exp\left(-\sum_p n_{pj}\gamma_{pz}\right)\end{aligned} \tag{2}$$

In equations (2): $\gamma_{iz} = - \ln r_{iz}$; $f_j = m_j + \sum_p n_{pj} - 1$;

$\nu_{ij} = \nu''_{is} - \nu'_{is}$, $n_{pj} = \nu'_{is}$ j = s, s = 1 ,...,m - for forward reactions;

$\nu_{ij} = \nu'_{is} - \nu''_{is}$, $n_{pj} = \nu''_{is}$ j = s + m, s = 1,...,m - for backward reactions;

$\nu_{ij} = \nu''_{is} - \nu'_{is}$, $n_{pj} = \nu'_{is}$ j = s + m, s = 1,...,(m + 1) ,..., (m+r) - for "mass exchange" reactions; m_j - the symbol of presence of catalytic element in chemical reaction; r_{iz} - molar concentration of specie i; i, p, q = 1, ... , n - species; and z - number of reactors. For above mentioned scheme of pressurization (z = 1, 2).

In adition to Eq. (2), the "equation of temperature" has been obtained (Dregalin *et al.*1985):

$$T - T_f - \frac{\sum_i (i\mu_i - I_{if}) r_i}{\sum C^f_{pi} r_i} = 0 \tag{3}$$

where I_{if} , C^f_{pi} - the values of specific enthalpy and specific heat at fixed values of temperature T_{if} .

The equations (2) and (3) are the basic for the description of high temperature processes in different power generation installations. For our specific case it is necessary to complement Equations (2) and (3) with different equations of conservation. The system of equations consists of those of mass and energy conservation. The equations of conservation of mass can be written as:

$$\frac{dM_1}{dt} = \dot{m}_g - \dot{m}_{1,2} - \dot{m}_d \,, \tag{4}$$

$$\frac{dM_2}{dt} = \dot{m}_{12} + \sum_{i=1}^{n} \dot{m}_{vi} - \sum_{i=1}^{n} \dot{m}_{ci} \tag{5}$$

where: $\dot{m}_g$ - flow rate of pressurized gas through the injector; $\dot{m}_d$ - flow rate of gas mixture through the drain valve; $\dot{m}_{12}$ - output of gas mixture flow rate from the zone 1 into zone 2;

$\sum_{i=1}^{n}\dot{m}_{vi}$ - flow rate of vapors from the free surface of the propellant; $\sum_{i=1}^{n}\dot{m}_{ci}$ - flow rate of condensed phase from the gas volume on the surface of the propellant; M_1 and M_2 - mass of gas mixture in first and second zones, correspondingly.

The equations of conservation of energy for zones 1 and 2 can be written in the form of conservation of enthalpy:

$$\frac{di_1}{dt}=\frac{\dot{m}_g(i_g-i_1)-Q_{w1}}{M_1} \qquad (6)$$

where: i_g, i_1 - the enthalpy of pressurized gas and gas mixture in the first zone, correspondingly; Q_{w1} - heat flux that was spent for the heating of the wall in zone 1;

$$\frac{di_1}{dt}=\frac{\dot{m}_{12}(i_1-i_2)+\sum_{i=1}^{n}\dot{m}_{vi}(i_{si}-i_2)-\sum_{i=1}^{n}\dot{m}_{ci}(i_{si}-i_2)-Q_{w2}-Q_h}{M_2} \qquad (7)$$

where: i_2 - enthalpy of gas mixture in zone 2; i_{si} - enthalpy of liquid propellant; Q_{w2} - heat flux that was spent for the heating of the wall in zone 2; Q_h - heat flux that was spent for heating the propellant.

In accordance with assumption of perfect gas the equations of perfect gas can be used for the description of gas state in zones 1 and 2:

$$M_1=P\mu_1/R_0T_1 \qquad (8)$$

$$M_2=P\mu_2(V-V_1)/R_0T_2. \qquad (9)$$

In addition to above mentioned equations, the specific equation of temperature stratification have been obtained:

$$T_2=\frac{T_w}{1+\frac{\ln(T_w/T_1+(1-T_w/T_1)\exp(-A_L))}{A_L}} \qquad (10)$$

where: T_w - average value of temperature of wall in zone 1; A_L - the value of range in formula of stratification of gas mixture temperature (Alemasov *et al.*, 1989).

The system of equations is closed by the equation of the changing of volume in the course of expulsion of propellant from the tank:

$$dV/dt=\dot{m}_p/\rho_p \qquad (11)$$

where: V - gas volume of tank; $\dot{m}_p$ - flow rate of propellant in the course of expulsion of propellant from the tank.

Equations (2) to (11) give the chance to predict unknown parameters of gas mixture in the tank: γ_{i1}, γ_{i2}, i_1, i_2, M_1, M_2, T_1, T_2, V, A_L, P. Several different schemes of gas flow had been created to predict the parameters of gas streams in the first zone of tank's free volume. These schemes are based on the theory of jets (Abramovitch, 1960). The model of prediction of heating of wall in first zone are based on the integral methods of theory of boundary layer. Mathematics for predicting mass and heat transfer between gas mixture and propellant combine the model of "thin film" (Frank – Kamenetski, 1987) and one - dimensional model of non-stationary heating of propellant (Dregalin *et al.*, 1985). Integration of Equations (2) to (11) are fulfilled using the finite - difference Pirumov's scheme and modified Newton's scheme. The described model of pressurization are oriented upon prediction of parameters of pressurization of tanks of sustainers of a launch vehicle.

4. NUMERICAL RESULTS

The comparison of numerical investigations results with experimental data (Polukhin *et al.*, 1987) demonstrated acceptable correlation. In both cases, a cylindrical tank with 10 m in heigth and 3 m in diameter was considered. One can see, for example, at Fig. 4 such comparison of values of pressure (P) and average temperature of gas mixture (T_{av}) for pressurization of liquid oxygen tank by gaseous helium (u_0 - the velocity of pressurized gas injection at a rate of 0.46 kg/s).

It would be interesting primarily to investigate the influence of finite values of rate of chemical reactions on the parameters of process of pressurization. All previous mathematical models (except the models of chemical pressurization) do not take into account chemical interaction among the species of gas mixture in the free volume of tank.

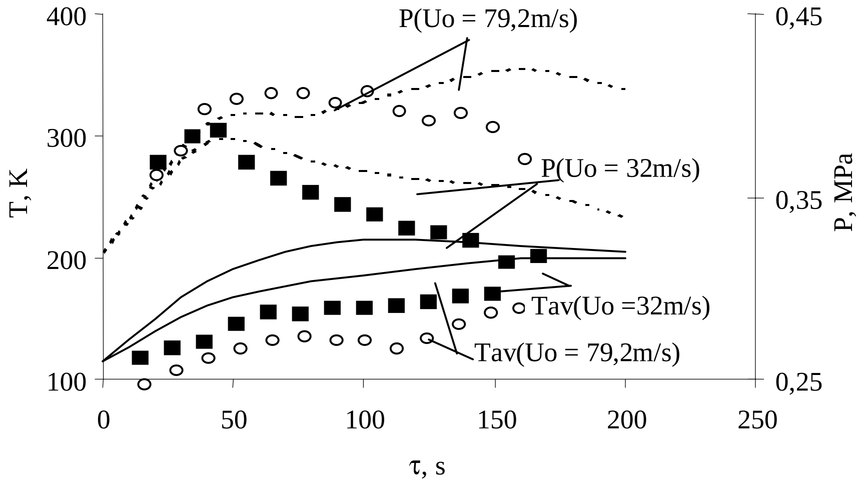

Figure 4- The comparison of experimental and numerical values of pressure of average temperature change.

To analyze the above-mentioned effect, following cases of pressurization of propellants were considered: hydrazine (N_2H_4) and nitrogen tetroxide (N_2O_4) by the products of gas generator that uses the fuel N_2H_4 and the oxidizer N_2O_4; the mass flow rate from the gas generator was 0.30 and 0.50 kg/s, respectively. One can see at Fig. 5 the plots of the change of pressure and average temperature in the free volume of tank with N_2H_4, in relation to time. The difference between the results of non-equilibrium (solid curve) and frozen (dotted curve), both for pressure and for average temperature, is equal approximately to 15%. The reason is inferred to heat effect of some reactions. Similar results have been obtained for the tank filled with N_2O_4.

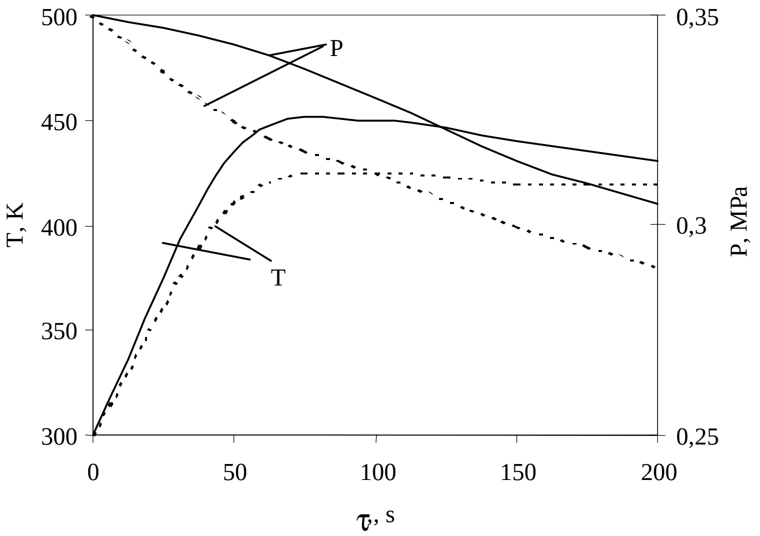

Figure 5- Comparison of “non-equilibrium” and “frozen” numerical results.
(——non-equilibrium;----frozen)

A great number of numerical investigations were devoted to the investigation of velocity of injection gas upon the pressure of the tank (Kozlov *et al.*, 1988). The reason of these investigations was the desire to reduce the quantity of gas required for pressurization.

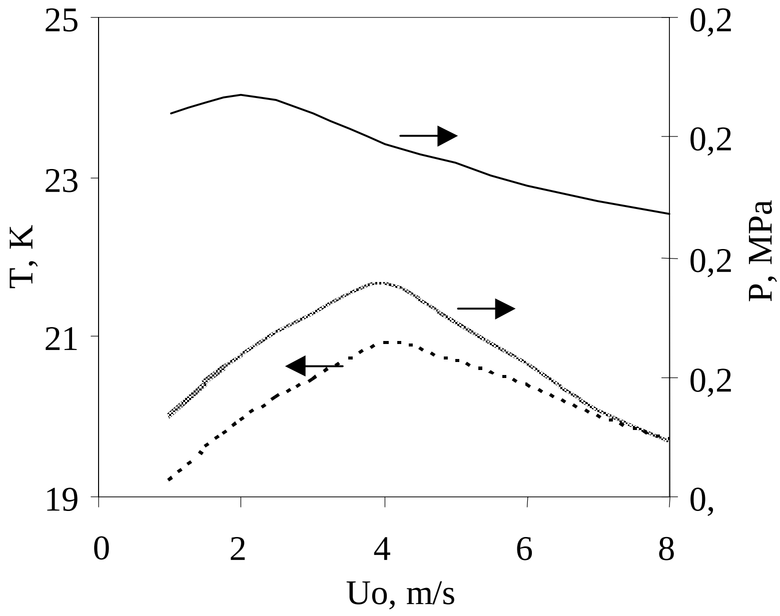

Figure 6- Influence of velocity of injected gas (oxygen tank).

To analyze the effect of injection velocity, the case of liquid oxygen pressurized by gaseous helium was also considered. One can see at Fig. 6 and 7 the plots of the average values of the pressure (p_{AV}), the plots of finite values of pressure (p_f) and the plots of average values of the temperature of gas mixture (T_{AV}) in the free volume in relation to different values of velocities of injected gas. Figure 6 presents the results for pressurization of oxygen tank and Fig. 7, the results for pressurization of nitrogen tetroxide tank.

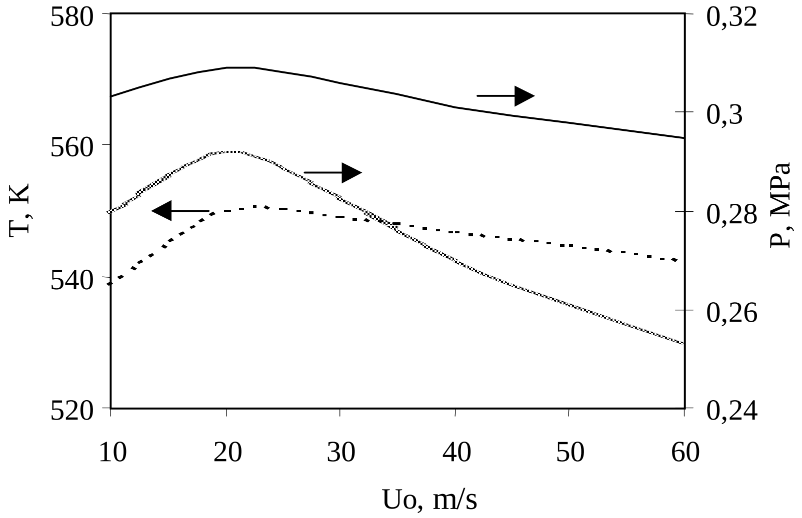

Figure 7- Influence of velocity of injected gas (nitrogen tetroxide tank).

All above-mentioned parameters have maximum values at the «optimal» values of velocity of injected gas. It is suggested that the reason of the maximum values of pressure and temperature at optimal velocities is due to the reduction of heat losses spent for the heating of tank's wall, to a minimum value. The numerical investigation of values of heat flux to the tank's walls confirm these fact. The optimum values of velocity have been obtained only for gas injector type (A), presented in Fig. 3.

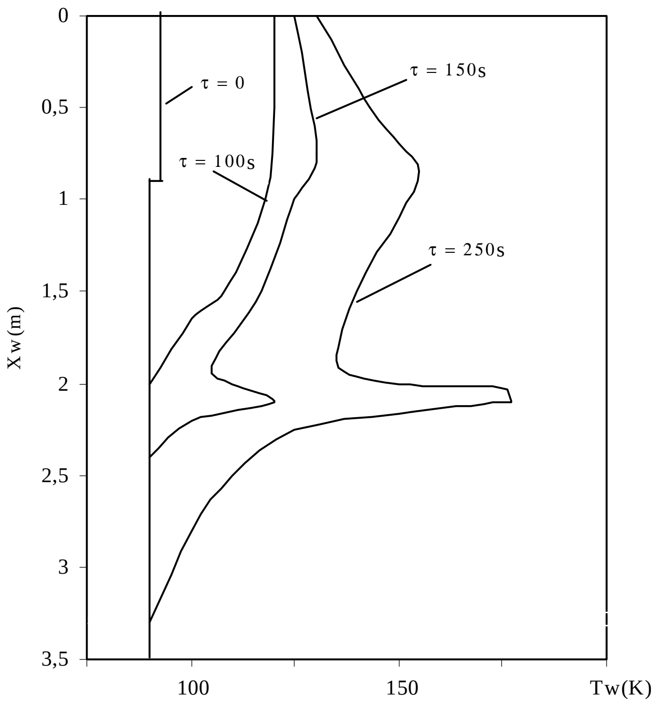

Figure 8- The profiles of temperature of tank's wall for injector of type (c).

Very useful results have been obtain in the course of pressurization of tank equipped with injector of type (c). The profiles of wall's temperature as a function of the time of pressurization of oxygen tank are presented in Fig. 8. Overheat can be observed in the area of incidence of high temperature gas jet on the tank's wall.

CONCLUSIONS

The presented model of propellant tank pressurization gave the chance to obtain several practically useful results in order to optimize the constructions of tanks and injectors, and to estimate the initial parameters of pressurizing gases. The main conclusions can be summarized as follows:

- Good correlation between numerical results and experimental data was obtained for pressure and average temperature temporal variations.
- The difference between non-equilibrium and frozen results for both pressure and average temperature was about 15%.
- There is an optimum injected gas velocity that can lead pressure and average temperature to the maximum values.

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REFERENCES

Abramovitch, G. N., 1960, Theory of turbulent jets, State Publishing House of Literature in Physics and Mathematics, Moscow,

Alemasov, V.Y., Kriukov, V. G., Dregalin, A. F. and Naoumov V. I., 1989, Simulation of high temperature processes in power generation systems, Nauka, Moscow,

Dregalin, A. F., Zenukov, I. A., Kriukov, V. G., Naoumov, V. I., 1985, Mathematical modeling of high- temperature processes in Power Plants, Kazan State University, Kazan.

Frank-Kamenetski, D. A., 1987, Diffusion and heat transfer in chemical kinetics, Nauka, Moscow.

Huzel, D. K. & Huang, D. H., 1992, Modern Engineering for Design of Liquid-Propellant Rocket Engines, AIAA, Washington, 1992.

Kozlov, A. A., Novikov, V. N. and Soloviev, E. V., 1988, Control and feed systems of liquid propellant rocket engine, Machinostroenie, Moscow.

Kriukov, V. G., Naoumov, V. I., Abdullin, A. L., Demin, A. V. and Trinos, T. V., 1997, Combustion and flow in propulsion and power generation systems, Yanus, Moscow.

Kudriatsev, B. M., 1983, Fundamentals of liquid propellant rocket engine theory and calculation, Vyschaia Chkola, Moscow.

Polukhin, D. A, Oreshenko, B. M. and Morov, V.A., 1987, Testing of hydro-pneumatic systems of satellites and space vehicles, Machinostroenie, Moscow.

Van Dresar, N. T., 1995, Prediction of pressurant mass requirements for axisymmetric liquid hydrogen tanks, 31st Joint Propulsion Conference, Paper AIAA-95-2964, San Diego, California.